

STATISTICS AND PROBABILITY:

- 1a) The probability that Amos goes to school on cloudy day is $\frac{7}{10}$ and $\frac{1}{5}$ on a clear day. If the probability of a cloudy day is $\frac{3}{5}$, find the probability that the day was cloudy given that he did not go to school.
- b) If P and Q are independent event;
- i) Show that P' and Q are also independent.
- ii) Find $P(Q)$ given that $P(P) = 0.25$ and $P(P \cup Q) = 0.75$.
2. During the national teachers' conference, 40% of the delegates supported the idea of the 10% salary increase for the financial year 2019. Given that journalists randomly interviewed 450 delegates, find the probability that;
- a) Less than 150 delegates supported the salary increase,
- b) Between 160 and 170 delegates inclusive supported the salary increase.
3. The table below shows the heights measured in cm for a group of senior six students;
- | Height | 177-186 | 187-191 | 192-196 | 197-201 | 202-206 | 207-216 |
|-----------|---------|---------|---------|---------|---------|---------|
| Frequency | 12 | 8 | 8 | 9 | 7 | 6 |
- a) Draw a histogram. Hence state the modal class.
- b) Calculate the (i) mean (ii) standard deviation. (ii) Mode
4. Events A , b and C are such that $P(A) = x$, $P(B) = y$ and $P(C) = x + y$. If $P(A \cup N) = 0.6$ and $P\left(\frac{B}{A}\right) = 0.2$,
- (i) Show that $4x + 5y = 3$.
- (ii) Given that B and C are mutually exclusive and that $P(B \cup C) = 0.9$, determine another equation in x and y .
- (iii) Hence find the values of x and y .
5. Of the 30 drivers interviewed, 9 have been involved in a car accident at some time. Of those who have been involved in the accident, 5 wear glasses. The probability of wearing glasses, given that the driver has not had a car accident is $\frac{1}{3}$. Find the probability that a;
- (a) Person chosen at random wears glasses.
- (b) Glasses wearer has been an accident victim.
- 6a) Given that A and B are two events such that $P(A') = 0.3$, $P(B) = 0.1$ and $P\left(\frac{A}{B}\right) = 0.2$. Find (i) $P(A \cup B)$ (ii) $P\left(\frac{A}{B'}\right)$

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- b) A box A contains 1 red, 3 green and 1 blue ball. Box B contains 2 red, 1 green and 2 blue balls. A balanced die is thrown and if the throw is a six, box A is chosen, otherwise box B is chosen. A ball is drawn at random from the chosen box. Given that a green ball is drawn, what is the probability that it came from box A?

7. The cumulative distribution function of a discrete random variable x is as shown in the table below.

x	1	2	3	4
$F(x)$	0.14	0.47	0.79	1.00

Find the (i) $P(2 < x \leq 4)$ (ii) median of x (iii) $P\left(\frac{x < 3}{2 \leq x < 4}\right)$ (iv) mean of x .

8. The probability density function of a continuous random variable x is given by;

$$f(x) = \begin{cases} \frac{2}{13}(x+1), & 0 \leq x \leq a \\ \frac{2}{13}(5-x), & a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}, \text{ find the values of } a \text{ and } b. \text{ Hence, sketch } f(x).$$

- (i) Calculate the mean and standard deviation of x .
(iii) Find $P(x < 2.5)$.

9. The Maths and Physics examination marks of a certain school are given in the following table.

Maths (x)	28	34	36	42	52	54	60
Physics(y)	54	62	68	70	76	68	74

- a)i) Plot the marks on the scatter diagram and comment on the relationship between the two subjects.
ii) Draw a line of best fit and use it to predict the Physics mark of a student whose Maths mark is 50.

- b) Calculate the rank correlation coefficient between the marks. Comment on the significance of Maths on Physics performance based on 5% level of significance

- 10a) A random sample of ten packets is taken. These have masses (Measured in kg) of $x_1, x_2, \dots, x_{10}$ such that; $\sum_{i=1}^{10} x_i = 2.57$ and $\sum_{i=1}^{10} x_i^2 = 0.6610$. Calculate a 95% confidence limits for the mean.

- b) The weights of ball bearings are normally distributed with mean 25gram and standard deviation 4 grams. If a random sample of 16 ball bearings is taken, find the probability that the mean of the sample is between 24.12 grams and 26.73 grams.

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11. The table below the number of reported accidents to workers in a certain factory over the past 12 years.

a) Find the mean and median of the workers

Age of Worker	15-	20-	25-	30-	35-	40- < 45
Number of employees	42	52	28	20	18	16

b) Plot a cumulative frequency curve and use it to estimate the;

- Proportion of employees who are not more than 27 years old.
- 80% central limits of the age of employees.

12. A survey of the mass (*kg*) of girls in the certain school in the final year was taken and the results are shown below;

Mass (<i>kg</i>)	< 45	<50	<60	<75	<80	<85	<90	<100	<105	<110
Cumulative frequency	0	9	25	46	63	77	82	89	93	94

- Display the information on a histogram and hence find the modal mass
- Calculate the standard deviation of the mass.

13. A random variable X has the probability density function

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

- Show that the variance of x is $\frac{(b-a)^2}{12}$.
- If the expected value and variance of the distribution is 1 and $\frac{3}{4}$ respectively,
 - $P(x < 0)$
 - Value of a such that $P(X > a + \sigma) = \frac{1}{4}$. Where σ is the standard deviation of X .

14. A pharmacist had the following records of unit price and quantities of drugs sold for the years 2010 and 2011.

Drug	Unit price per carton		Quantities per carton	
	2010	2011	2010	2011
Aspirin	80	125	40	45
Panadol	100	90	70	90
Quinine	55	75	8	10
Coartem	90	100	10	10

Taking 2010 as the base year,

- Calculate the price relatives for each drug in 2011.
- Obtain the simple aggregate price index number for 2011.

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- (iii) Calculate the weighted price index and comment on it.
- (iv) Calculate the weighted aggregate price index and comment on it.

MECHANICS:

15. A body of mass 0.2kg is acted upon by a force $\mathbf{F} = 8t \mathbf{i} - 4t^2 \mathbf{j} + 2(3 - t^2) \mathbf{k} \text{ N}$. Initially, the body is at rest at a point with position vector $-10 \mathbf{i} + 12 \mathbf{j} - 4 \mathbf{k}$. Find the
- (i) velocity after time t seconds,
 - (ii) Distance covered by the body in 2seconds.
16. An object of mass 4kg moves under the action of three forces $\mathbf{F}_1 = \begin{pmatrix} \frac{t}{2} - 1 \\ t - 3 \end{pmatrix} \text{ N}$, $\mathbf{F}_2 = \begin{pmatrix} \frac{t}{2} + 2 \\ \frac{t}{2} - 4 \end{pmatrix} \text{ N}$ and $\mathbf{F}_3 = \begin{pmatrix} t - 3 \\ 3t - 2 \end{pmatrix} \text{ N}$ at time t seconds. Find the
- (i) Work done by the resultant force when the object is displaced from point $A(4, 5)$ to a point $B(7, -9)$ within a time of 1 s .
 - (ii) Size of the acceleration of the object after a time of 2 s .
 - (iii) Power developed when the object moves with velocity of $3\mathbf{i} - 6\mathbf{j} \text{ m s}^{-1}$ after a time of 2 s .
17. Two smooth inclined planes meet at right angles, the inclination of one to the horizontal being 30° and the other being 60° . Bodies of masses 2kg and 4kg lie on the respective planes, and the two masses are joined by a light inextensible string passing over a smooth fixed pulley at the intersection of the planes.
- (i) If the masses are released from rest, find the acceleration of the masses and tension of the string.
 - (ii) If the surface with 4kg mass experiences a frictional force of 0.25N, find the new acceleration of the masses when the system is set from rest.
18. Four forces represented by $i - 4j$, $3i + 6j$, $-9i + j$ and $5i - 3j$, at a points with position vectors $3i - j$, $2i + 2j$, $-i - j$ and $-3i + 4j$ respectively.
- (a) Show that the forces reduce to a couple and find its moment
 - (b) If the fourth force is removed, find the magnitude and direction of the resultant forces.
- 19a) If the horizontal range of a particle projected with a velocity v is a , show that the greatest height x , attained is given by the equation $16gx^2 - 8v^2x + ga^2 = 0$, where g is acceleration due to gravity.
- b) A small ball is projected from a point A on a level ground and passes at a height of 2metres over a point on the ground at a distance of 12m from A. The ball is caught at a height of 1metre by a girl who is 24metres from A. Find the angle of projection of the ball as it leaves point A. ($g=10\text{m/s}^2$)

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- 20a) A particle of mass 2kg is pushed up a rough inclined plane whose slope is 3 in 5 by a horizontal force of 46.25N. The particle starts from rest at P and is moved up a line of greatest slope of the plane to the point Q, where PQ=15m. The coefficient of friction between the plane and the particle is 0.25. Find the speed acquired by the particle at Q (Take $g=10\text{ms}^{-2}$)
- b) A pump draws water from a tank and delivers to a height of 10m at the end of a horse. The cross-sectional area of the horse is 10cm^2 and the water leaves the end of the horse with a speed of 7.5 m s^{-1} . Find the rate at which the pump is working.
- 21a) A particle is projected at an angle of 30° with a speed of 21 m s^{-1} . If the point of projection is 5m above the horizontal grounds, find the horizontal distance that the particle travels before striking the ground.
- b) A boy throws a ball at an initial speed of 40 ms^{-1} at an angle of elevation α . Show that the times of flight corresponding to the horizontal range of 8 m are positive roots of the equation $T^4 - 64T^2 + 256 = 0$. (Take $g=10\text{ ms}^{-2}$).
22. A light inextensible string has one end attached to a ceiling. The string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley. Particle of mass 5 kg is attached at the free end of the string. The sections of the string not in contact with the pulleys are vertical. If the system is released from rest and moves in a vertical plane, find the:
- Acceleration of the system.
 - Tension in the string.
 - Distance moved by the moveable pulley in 1.5 s.
- 23a) A body of mass 5 kg slides a distance of 8 m down a rough plane inclined at an angle of $\sin^{-1}\left(\frac{4}{5}\right)$ to the horizontal. If the coefficient of friction is 0.4, find the velocity attained by the body.
- b) A particle of mass 50 kg is suspended by two light inelastic strings of lengths 9 m and 12m attached to two points distant 15 m apart. Calculate the tensions in the strings.
24. A rod AB 1m long has a weight of 20N acting at a point 60cm from A. It rests horizontally with A hinged on a vertical wall. A string BC is fastened to the wall at C, 75cm vertically about A find the;
- Tension in the string
 - Reaction from at point A.
- 25a) A particle travelling in a straight line with the constant acceleration covers distances x and y in the 3rd and 4th seconds of its motion respectively. Show that its initial speed is given by $U = \frac{1}{2}(7x - 5y)$.

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- b) Two stations A and B are a distance of $6x$ m apart along a straight path. A train starts from rest at A and accelerates uniformly until it attains a speed of $V \text{ ms}^{-1}$ covering a distance of x m. The train then maintains this speed until it has travelled a further $3x$ m. It then retards uniformly to rest at B. show that if T is the time taken for a train to travel from A to B then $T = \left(\frac{9x}{V}\right) \text{ s}$
26. Forces of magnitudes **6N, 6N, 4N, 10N** and **8N** act along **AB, BC, CD, DB** and **AD** respectively, in the directions indicated by the order of the letters of a rectangle **ABCD** of dimensions 4m by 3m. Find the;
- Magnitude of their resultant
 - Equation of the line of action of their resultant force
 - Distance from **B** where it cuts **AB**.
- 27a) A, B, C and D are the points (0,0), (10,0), (7,4) and (3,4) respectively. If AB, BC, CD and DA are made of a thin wire of uniform mass. Find the coordinates of the centre of gravity.
- b)i) If instead ABCD is a uniform lamina, find its centre of gravity G
- ii) If the lamina hung from B, find the angle AB makes with the vertical
28. An object of mass 5kg is initially at rest at a point whose position vector is $-2\mathbf{i} + \mathbf{j}$. If it is acted upon by a force, $\mathbf{F} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, find
- The acceleration
 - The velocity after 3s
 - Its distance from the origin after 3s.
- 29a) A car is being driven at 20 m s^{-1} on a bearing of 040° . the wind is blowing from 330° with a speed of 10 m s^{-1} . Find the velocity of the wind as experienced by the driver of the car.
- b) At 8a.m two boats A and B are 5.2km apart with A due west of B, and B travelling in a direction $\text{N}20^\circ\text{W}$ at a steady speed of 13 km h^{-1} . If A travels due north at 12 km h^{-1} , determine;
- The path of B relative to A
 - The distance between A and B at 8:30am.
30. A particle of mass $M \text{ kg}$ is attached to a string of natural length L and modulus of elasticity $4Mg$. The other end is attached to a fixed point O. If the particle is released from rest at O.
- Find the greatest distance below O reached by the particle and its acceleration at this point.
 - Show that the speed of the particle is $1.5\sqrt{gL}$ when the acceleration is instantaneously zero.

NUMERICAL METHODS:

31a) The table below shows x and f(x);

x	50.24	48.11	46.93	44.06
f(x)	4.116	7.621	9.043	11.163

Use linear interpolation or extrapolation to estimate;

(i) x when f(x)=8.614

(ii) $f^{-1}(5.107)$

b) Use trapezium rule with six ordinates to find the value of $\int_1^2 \frac{x^2}{x^2 + 1} dx$, correct to three decimal places.

c) Find the exact value of $\int_1^2 \frac{x^2}{x^2 + 1} dx$, correct to three decimal places. How can you improve on the degree of accuracy?

32a) Use the trapezium rule with 6 strips to evaluate $\int_1^2 (x-1) \ln x dx$ correct to 4 decimal places.

b) Find the exact value of $\int_1^2 (x-1) \ln x dx$. Hence find the maximum possible error in your calculations in (a) above.

33. If the numbers x and y are approximations with errors Δx and Δy respectively. Show that the maximum absolute error in the approximations of $x^2 y$ is given by

$|2xy\Delta x| + |x^2\Delta y|$ Hence find the limits within which the true value of $x^2 y$ lies given that $x = 2.8 \pm 0.016$ and $y = 1.44 \pm 0.008$

34a) Show that the equation $3x^2 - x - 5 = 0$ has a real root between $x = -1.4$ and $x = -1.2$.

b) Use linear interpolation to estimate the initial root

c) Using the root in (b), use Newton – Raphson Method to find the root, correct to two decimal places.

35a) The dimensions of a rectangle are 8cm and 4.26.

(i) State the maximum possible error in each dimension.

(ii) Find the range within which the area of the rectangle lies. (correct to 2 decimal places)

b) The radius r and height h of a cylinder are measured with corresponding errors Δr and Δh respectively. Show that the maximum possible error in the volume is

$$\left| \frac{\Delta h}{h} \right| + 2 \left| \frac{\Delta r}{r} \right|$$

36a) Derive the simplest formula based on Newton Raphson's method to show that for

the equation $3x = \ln 3$ it satisfies $x_{r+1} = \frac{1}{3} \left\{ \frac{e^{3x_r} (3x_r - 1) + 3}{e^{3x_r}} \right\}$

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- b) Draw a flow chart that;
 - i) reads the initial approximation x_0 ,
 - ii) Accepts estimations to 4 significant figures
 - iii) Prints the root to 4sf .
 - iv) Perform a dry-run for your flow chart for $x_0 = \frac{1}{3}$.

37a) Show that the iterative formula based on Newton Raphson's Method for approximating the root of the equation $2\ln x = x - 1$ is given by;

$$x_{n+1} = x_n \left(\frac{2 \ln x_n - 1}{x_n - 2} \right) \quad n = 0, 1, 2, \dots$$

- (b) Draw a flow chart that,
 - (i) Reads the initial approximation x_0 of the root.
 - (ii) Computes and prints the root correct to two decimal places, using the formula in (a) above.

END